A COURSE SYLLABUS – DOCTORAL SCHOOL
regarding the qualification cycle from 2024/2025 to 2027/2028

GENERAL INFORMATION ABOUT COURSE	
Course title	*OPTIONAL SPECIALIST SUBJECT:* *Haar measure and its properties.*
Name of the unit running the course	Doctoral School at University of Rzeszów
Type of course *(obligatory, optional)*	compulsory - optional specialist subject
Year and semester of studies	year II, semester IV
Discipline	mathematics
Language of Course	Polish/English language
Name of Course coordinator	**Prof. dr hab. Mykhaylo Zarichnyy**
Name of Course lecturer	**Prof. dr hab. Mykhaylo Zarichnyy**
Prerequisites	Real analysis, measure theory, Algebra, group theory, Functional analysis, Topology

BRIEF DESCRIPTION OF COURSE
(100-200 words)

Haar measure assigns an 'invariant volume' to subsets of locally compact topological groups, consequently defining an integral for functions in these groups. Haar measures are used in many areas of analysis, number theory, group theory, representation theory, statistics, probability theory and ergodic theory. There will be a discussion of measures in metric spaces, with particular emphasis on Borel measures. A brief introduction of the theory to topological groups will be given, in particular it will be proved that every topological (Hausdorff) group is completely regular and if it satisfies the first axiom of countability (having a countable basis for open sets at every point), then it is metrizable, moreover with a left invariant metric.

Next, a Banach proof of the existence of an invariant measure on a compact metrizable topological group will be presented, followed by a von Neumann proof of the existence and uniqueness of a normed Haar measure on any compact topological group. Some applications of Haar measures will be presented.

COURSE LEARNING OUTCOMES AND METHODS OF EVALUATING LEARNING OUTCOMES

Learning outcome	The description of the learning outcome defined for the course	Relation to the degree programme outcomes (symbol)	Learning Format (Lectures, classes,...)	Method of assessment of learning outcomes (e.g. test, oral exam, written exam, project,...)
Knowledge (no.)	knows and understands, has knowledge			
P8S_WG/1	has a broad theoretical knowledge and is familiar with current scientific developments including worldwide in Haar measure theory and general issues in related chapters.	P8S_WG	Conversatory	Examination/ written work
P8S_WG/2	the development trend and the latest research achievements in Haar measure theory, including worldwide.	P8S_WG	Conversatory	Examination/ written work
P8S_WG/3	has interdisciplinary knowledge, knows and understands the scientific terminology used in his/her discipline in the native and foreign languages.	P8S_WG	Conversatory	Examination/ written work

P8S_WK/1	knows and understands the impact of the development of mathematical sciences on the progress of civilisation.	P8S_WK	Conversatory	Examination/ written work
Skills (no.)	can			
P8S_UW/1	identify and solve a research problem, define a research objective, formulate a hypothesis and subject of research, develop techniques, methods and research tools and to make inferences on the basis of research results.	P8S_UW	Conversatory	Examination/ written work
P8S_UW/2	select and use the available scientific literature to diagnose and solve research problems and innovative activities related to related to their scientific work and to apply the relevant workflow to the creation of new scientific output.	P8S_UW	Conversatory	Examination/ written work
P8S_UW/3	use their broad scientific knowledge to analyse and evaluate research results, forming opinions, including critical judgements, on this basis.	P8S_UW	Conversatory	Examination/ written work
P8S_UK/6	is able to carry out research work based on foreign-language literature, is able to speak in public to present the results of scientific research and to participate in participate in discussions on scientific and professional topics in an international and professional topics in an international environment.	P8S_UK	Conversatory	Examination/ written work
Social competence (no.)	is ready to			
P8S_KK/3	solve cognitive and practical problems related to measurement theory with the help of his/her knowledge.	P8S_KK	Conversatory	Examination/ written work

Semester (no.)	Lectures	Seminars	Conversatory / Lab classes	Internships	others	ECTS
II	-	-	15 hrs.	-	-	2

METHODS OF INSTRUCTION

- CONVERSATIONS/LABORATORIES IN TRADITIONAL FORM;
- CLASSES WITH MULTIMEDIA PRESENTATION;
- PROJECT;
- DISCUSSION.

COURSE CONTENT

Conversation:

1. Lebesgue measure in Euclidean space.
2. measures in metric spaces.
3. topological groups.
4. Banach limits and Haar measure.
5. Haar measure on compact groups.
6. applications of Haar measure.
7. Haar measure on locally compact groups.

COURSE ASSESSMENT CRITERIA

The examination takes place in written form after each semester of the course (semester 4).
The doctoral student prepares a written paper on issues related to the subject of the course.

In order to obtain a positive grade, a conversion factor for the corresponding percentage of points obtained is applied:
- up to 50% - insufficient, (the written work does not meet the criteria)
- 51% - 60% - satisfactory, (the written work contains a narrow range of knowledge, supported by basic literature for the subject)
- 61% - 70% - satisfactory plus, (the written work contains a range of knowledge, supported by basic literature for the subject);
- 71% - 80% - good, (written work contains a range of knowledge, supported by primary and secondary literature for the subject);
- 81% - 90% - good plus, (written work contains a comprehensive range of knowledge, supported by basic and supplementary literature for the subject;
- 91% - 100% - very good (the written work contains an extensive range of knowledge, supported by literature beyond the obligatory literature for the subject).

TOTAL PhD STUDENT WORKLOAD REQUIRED TO ACHIEVE THE INTENDED LEARNING OUTCOMES – NUMBER OF HOURS AND ECTS CREDITS

Activity	Number of hours
Scheduled course contact hours	15 hrs
Other contact hours involving the teacher (consultation hours, examinations)	5
Non-contact hours – student`s own work (preparation for classes or examinations, project, etc.)	40
Total number of hours	60
Total number of ECTS credits	**2 ECTS**

INSTRUCTIONAL MATERIALS

Compulsory literature:	J. DIESTEL, A. SPALSBURY, THE JOYS OF HAAR MEASURE, GRADUATE STUDIES IN MATHEMATICS, VOL. 150, PROVIDENCE, RI: AMERICAN MATHEMATICAL SOCIETY, (2014), ISBN 978-1-4704-0935-7.
	LYNN A. LOOMIS, AN INTRODUCTION TO ABSTRACT HARMONIC ANALYSIS, UNIVERSITY SERIES OF UNIVERSITY SERIES OF HIGHER MATHEMATICS, VAN NOSTRAND, PRINCETON N.J. 1953.
	PAUL R. HALMOS, MEASURE THEORY. NEW YORK: SPRINGER SCIENCE+BUSINESS MEDIA. (1950). ISBN 978-1-4684-9442-6.
	L. NACHBIN, THE HAAR INTEGRAL, PRINCETON, NJ: D. VAN NOSTRAND (1965).
Complementary literature:	Donald L. Cohn, Measure theory, Birkhäuser, 1997. ISBN 3-7643-3003-1.
	N. Bourbaki, Integration II, Berlin-Heidelberg-New York: Springer (2004).

*(1 ECTS CREDIT CORRESPONDS TO 25 - 30 HOURS OF THE TOTAL WORKLOAD OF A DOCTORAL STUDENT, NEEDED TO ACHIEVE THE ESTABLISHED EFFECTS).

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Date and signature of the Course lecturer

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  Approved by the Head of the Department or an authorised person