Appendix No. 1.5 to the Resolution No. 61/2025
of the Rector of the University of Rzeszów

SYLLABUS

REGARDING THE QUALIFICATION CYCLE FROM 2026 TO 2029
ACADEMIC YEAR 2027/2028

1. BASIC COURSE/MODULE INFORMATION

Course/Module title	Algebra with number theory
Course/Module code *	
Faculty (name of the unit offering the field of study)	Faculty of Exact and Technical Sciences
Name of the unit running the course	Institute of Mathematics
Field of study	Mathematics
Qualification level	First-cycle studies (Bachelor's)
Profile	General academic
Study mode	Full-time
Year and semester of studies	Year 2, Semester 3
Course type	Major subject
Language of instruction	English
Coordinator	Prof. Mykhaylo Zarichnyy, PhD, DSc
Course instructor	Prof. Mykhaylo Zarichnyy, PhD, DSc

* - as agreed at the faculty

1.1. Learning format – number of hours and ECTS credits

Semester (no.)	Lectures	Classes	Laboratories	Seminars	Practical classes	Internships	others	**ECTS credits**
3	30	30						6

1.2. Course delivery methods

☒ conducted in a traditional way

☐ involving distance education methods and techniques

1.3. Course/Module assessment (exam, pass with a grade, pass without a grade)

Lecture – exam, classes – pass with a grade

2. PREREQUISITES

Knowledge of logic, set theory and linear algebra.

3. Objectives, Learning Outcomes, Course Content, and Instructional Methods

3.1. Course/Module objectives

O1	To acquire knowledge of the basic concepts and theorems of algebra and number theory.
O2	To develop the ability to recognise algebraic structures in problems from various branches of mathematics.
O3	To acquire the ability to use the basic concepts and tools of abstract algebra and number theory.
O4	To develop the ability to conduct algebraic and number-theoretic reasoning.

3.2. Course/Module Learning Outcomes

Learning Outcome	The description of the learning outcome defined for the course/module	Relation to the degree programme outcomes
LO_01	The student knows and understands the basic concepts, theorems and methods of abstract algebra in the field of group theory, ring theory and number theory, and also knows examples illustrating specific concepts in this area.	K_W02, K_W03, K_W04,
LO_02	The student can apply the acquired concepts, theorems and methods of algebra and number theory to solve specific problems. The student recognises the possibility of using abstract algebra tools to solve mathematical or practical problems.	K_U01, K_U13
LO_03	The student can present correct mathematical reasoning concerning concepts in the field of algebra and number theory in a comprehensible manner, and can carry out proofs of selected theorems from the theory of groups, rings and fields.	K_U02
LO_04	The student can identify and demonstrate the presence of a group or ring structure in various mathematical problems. The student recognises substructures (subgroups, subrings, subfields) of given algebraic structures. The student constructs new groups and rings using Cartesian products and investigates the basic properties of groups, rings and their elements. The student knows the basic properties of integers in number theory.	K_U01, K_U13, K_U05
LO_05	The student uses the concepts of a normal subgroup of a group and an ideal of a ring. The student constructs quotient groups and quotient rings and investigates their properties.	K_U01, K_U13, K_U05

LO_06	The student works with the concepts of homomorphism of algebraic structures (groups, rings), kernel and image of a homomorphism. The student understands the significance of isomorphisms and recognises isomorphic structures.	K_U01, K_U13
LO_07	The student uses the basic concepts of divisibility theory in integral domains, in particular distinguishes special types of elements. The student applies appropriate methods to find the greatest common divisor and the least common multiple.	K_U01, K_U13
LO_08	The student can formulate questions aimed at understanding the studied problem and can express their own opinions on theoretical and practical issues in algebra and number theory.	K_K02

3.3. Course content

A. Lectures

Content outline
Operations and their properties. Groups, transformation groups, permutation groups. Subgroups, cosets, normal subgroups, quotient groups.
Group homomorphisms, theorems concerning group homomorphisms (isomorphisms).
Direct sum of groups. Cyclic groups. Finitely generated Abelian groups.
Rings, integral domains, fields. Subrings, ideals, quotient rings.
Ring homomorphisms, theorems concerning ring homomorphisms (isomorphisms).
Characteristic of an integral domain. Field of fractions of an integral domain.
Polynomials, polynomial rings. Roots of a polynomial, multiple roots. Polynomial division with remainder. Polynomials in several variables (for information).
Divisibility relation in an integral domain, the association relation, group of invertible elements. Factorisation of an element, reducible and irreducible elements, prime elements. Rings with factorisation, unique factorisation domains. Greatest common divisor and least common multiple.
Relation between the greatest common divisor and the least common multiple.
Coprime numbers, the fundamental theorem of arithmetic, the Euclidean algorithm.
Calculating the greatest common divisor and the least common multiple of any finite set of natural numbers.
The Chinese remainder theorem, Thue's theorem.

B. Classes, laboratories, seminars, practical classes

Content outline
Investigating properties of operations. Groups, Abelian groups. Transformation groups, permutation groups. Subgroups. Cosets relative to a subgroup. Normal subgroups. Constructing quotient groups.
Group homomorphisms, types of homomorphisms. Kernel and image of a homomorphism. Determining whether groups are isomorphic.
Finding the orders of group elements. Properties of cyclic groups, investigating the cyclicity of groups.
Rings, commutative rings, rings with unity. Zero divisors and invertible elements. Integral domains. Fields. Subrings and ideals. Quotient rings. Prime ideals and maximal ideals.
Ring homomorphisms, kernel and image of a homomorphism. Determining whether rings (fields) are isomorphic. Characteristic of an integral domain.
Polynomial rings. Polynomial division with remainder. Roots of polynomials and their multiplicities.
Divisibility and association relations in an integral domain. Finding divisors of an element of an integral domain and elements associated with it. Reducible and irreducible elements. Prime elements. Factorisation of an element into irreducible factors, uniqueness of factorisation into irreducible factors. Greatest common divisor and least common multiple. Sum and product of ideals, principal ideals in divisibility theory. Applying the Euclidean algorithm.
Coprime numbers, the fundamental theorem of arithmetic, the Euclidean algorithm.
Calculating the greatest common divisor and the least common multiple of any finite set of natural numbers.

3.4. Methods of Instruction

Lectures: traditional lecture.
Classes: problem solving, discussion, group work.

4. Assessment techniques and criteria

4.1. Methods of evaluating learning outcomes

Learning outcome	Methods of assessment of learning outcomes (e.g. test, oral exam, written exam, project, report, observation during classes)	Learning format (lectures, classes,...)
LO_01	test, observation during classes, written exam	classes, lectures

LO_02	test, observation during classes, written exam	classes, lectures
LO_03	test, observation during classes, written exam	classes, lectures
LO_04	test, observation during classes, written exam	classes, lectures
LO_05	test, observation during classes, written exam	classes, lectures
LO_06	test, observation during classes, written exam	classes, lectures
LO_07	test, observation during classes, written exam	classes, lectures
LO_08	test, observation during classes	classes, lectures

4.2. Course assessment criteria

Class credit is awarded on the basis of written tests and class participation.
The condition for obtaining class credit is achieving at least 50% of points from each test. The final grade for classes is then determined according to the following scale:

below 50% pts. – fail,
[50 – 60%) pts. – satisfactory,
[60 – 70%) pts. – satisfactory plus,
[70 – 80%) pts. – good,
[80 – 90%) pts. – good plus,
[90 – 100%] pts. – very good.

Class participation may raise the grade by at most half a grade.
Lecture credit is awarded on the basis of a test during the lecture.

Examination: The condition for taking the examination is obtaining credit for lectures and classes. The condition for passing is achieving at least 50% of the points. The final examination grade is determined according to the following scale:

below 50% pts. – fail,
[50 – 60%) pts. – satisfactory,
[60 – 70%) pts. – satisfactory plus,
[70 – 80%) pts. – good,
[80 – 90%) pts. – good plus,
[90 – 100%] pts. – very good.
The condition for passing the resit examination is achieving at least 50% of the points.
The final grade for the resit examination is determined according to the same scale.

5. Total student workload needed to achieve the intended learning outcomes – number of hours and ECTS credits

Activity	Number of hours
Course hours	60
Other contact hours involving the teacher (consultation hours, examinations)	5
Non-contact hours - student's own work (preparation for classes or examinations, projects, etc.)	85
Total number of hours	150
Total number of ECTS credits	6

* One ECTS point corresponds to 25-30 hours of total student workload

6. Internships related to the course/module

Number of hours	*Not applicable*
Internship regulations and procedures	*Not applicable*

7. Instructional materials

Compulsory literature:

1. David M. Burton, Elementary Number Theory, Seventh Edition, 2011 file:///C:/Users/Ewa/Downloads/david-m-burton-elementary-number-theory-mcgraw-hill-education-2010.pdf
2. David S. Dummit, Richard M. Foote, Abstract Algebra, Third edition, Willey, 2011.

Complementary literature:

1. Lindsay N. Childs, A Concrete Introduction to Higher Algebra, Springer, 2009.

Approved by the Head of the Department or an authorised person