

SYLLABUS

REGARDING THE QUALIFICATION CYCLE FROM 2026 TO 2029

ACADEMIC YEAR 2026/2027.

1. BASIC COURSE/MODULE INFORMATION

Course/Module title	Linear Algebra with Geometry 2
Course/Module code*	
Faculty (name of the unit offering the field of study)	Faculty of Exact and Technical Sciences
Name of the unit running the course	Institute of Mathematics
Field of study	Mathematics
Qualification level	First-cycle studies (Bachelor's)
Profile	General academic
Study mode	Full-time
Year and semester of studies	Year 1, Semester 2
Course type	Major subject
Language of instruction	English
Coordinator	Ewa Rak, PhD
Course instructor(s)	

* - as agreed at the faculty

1.1. Learning format – number of hours and ECTS credits

Semester (no.)	Lectures	Classes	Laboratories	Seminars	Practical classes	Internships	others	ECTS credits
2	30	30						6

1.2. Course delivery methods

- conducted in a traditional way
- involving distance education methods and techniques

1.3. Course/Module assessment (exam, pass with a grade, pass without a grade)

- Lectures – exam,
- Classes – pass with a grade

2. PREREQUISITES

Knowledge of mathematics at the level of the basic secondary school leaving examination, linear algebra and geometry 1, calculus 1.

3. OBJECTIVES, LEARNING OUTCOMES, COURSE CONTENT, AND INSTRUCTIONAL METHODS

3.1. Course/Module objectives

O1	To introduce the fundamental concepts of linear algebra.
O2	To introduce the basic concepts of analytic geometry.
O3	To introduce the core proof methods used in linear algebra and analytic geometry.
O4	Familiarize students with the basic computational techniques used in linear algebra and analytic geometry.

3.2. COURSE/MODULE LEARNING OUTCOMES

Learning Outcome	The description of the learning outcome defined for the course/module	Relation to the degree programme outcomes
LO_01	The student defines classical concepts and formulates basic theorems in the field of linear algebra and analytic geometry.	K_Wo1, K_Wo2, K_Wo3, K_Wo4
LO_02	The student has knowledge of proof methods used in linear algebra and analytic geometry.	K_Wo2, K_Wo4
LO_03	The student has knowledge of methods for solving problems in linear algebra and analytic geometry.	K_Wo3
LO_04	The student proves basic theorems in the field of linear algebra and analytic geometry.	K_Uo1
LO_05	The student finds the matrices of linear transformations in different bases.	K_Uo1, K_Uo2, K_Uo7
LO_06	The student calculates eigenvalues and eigenvectors of a matrix and can explain the geometric meaning of these concepts;	K_Uo1, K_Uo2, K_Uo7
LO_07	The student can write various equations of lines and planes and examine their mutual position.	K_Uo1, K_Uo2, K_Uo8
LO_08	The student applies the properties of conic sections in problem solving.	K_Uo1, K_Uo2, K_Uo8
LO_09	The student can write equations of surfaces of revolution.	K_Uo1, K_Uo2, K_Uo8
LO_10	The student is ready to critically evaluate the content acquired in the field of linear algebra and analytic geometry and to recognise the need to improve their own competencies in this area.	K_Ko1
LO_11	The student is ready to adopt a critical stance towards content received in the field of linear algebra and analytic geometry in terms of its logical justification.	K_Ko2

LO_12	The student is ready to ask questions aimed at understanding the studied problem in the field of linear algebra and analytic geometry when solving theoretical and practical mathematical issues.	K_K03
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3.3. Course content

A. Lectures

Content outline
Linear transformations: definition of a linear transformation, kernel and image of a linear transformation, rank of a linear transformation, monomorphism, epimorphism, isomorphism. Matrix representation of a linear transformation. Matrix multiplication and composition of linear transformations. Transition matrix, matrix of a linear transformation after a change of basis.
Endomorphisms: eigenvalue and eigenvector of an endomorphism, characteristic polynomial.
Quadratic forms: bilinear mapping, matrix and rank of a bilinear mapping, diagonalisation of a quadratic form.
Euclidean vector spaces: inner product, norm determined by the inner product, Schwarz inequality, orthonormal basis, orthogonal matrix.
Vectors: operations on vectors in \mathbb{R}^3 : addition, subtraction, scalar multiplication, dot product, cross product.
Analytic geometry in \mathbb{R}^2: Line in the plane. Definitions and equations of conic sections.
Analytic geometry in \mathbb{R}^3: Line and plane in space.

B. Classes, laboratories, seminars, practical classes

Content outline
Linear transformations: checking the linearity of a transformation, finding the kernel and image of a linear transformation.
Matrix of a linear transformation: finding the transformation matrix in different bases, determining and using the matrix representation of a linear transformation.
Operations on linear transformations: addition, scalar multiplication, composition; relation to transformation matrices.
Transformation matrix after a change of basis, transition matrix.
Endomorphisms: eigenvalue and eigenvector of an endomorphism, characteristic polynomial.
Quadratic forms: checking bilinearity of a mapping, finding the matrix and rank of a bilinear mapping, expressing a quadratic form in diagonal form.
Euclidean vector spaces: investigating and applying properties of the inner product, calculating the norm determined by the inner product, applying the Schwarz inequality, checking the orthogonality of matrices.
Vectors: addition, subtraction, scalar multiplication, dot product, cross product in \mathbb{R}^3 ; applying vector operations to problem solving.
Analytic geometry in \mathbb{R}^2: Line in the plane, various types of line equations, mutual position of lines.

Analytic geometry in R^3 : Lines and planes in space, various types of equations, investigating the mutual position of two lines, a line and a plane, and two planes. Axial and planar symmetry.

3.4. Methods of Instruction

Lectures: traditional lecture with multimedia elements

Classes: problem analysis with discussion, problem solving

4. Assessment techniques and criteria

4.1. Methods of evaluating learning outcomes

Learning outcome	Methods of assessment of learning outcomes (e.g. test, oral exam, written exam, project, report, observation during classes)	Learning format (lectures, classes,...)
LO_01	test, exam, observation during classes	lectures, classes
LO_02	exam, observation during classes	lectures, classes
LO_03	test, exam, observation during classes	lectures, classes
LO_04	exam, observation during classes	lectures, classes
LO_05	test, exam, observation during classes	lectures, classes
LO_06	test, exam, observation during classes	lectures, classes
LO_07	test, exam, observation during classes	lectures, classes
LO_08	test, exam, observation during classes	lectures, classes
LO_09	test, exam, observation during classes	lectures, classes
LO_10	observation during classes	lectures, classes
LO_11	observation during classes	lectures, classes
LO_12	observation during classes	lectures, classes

4.2. Course assessment criteria

Class credit is awarded on the basis of written tests (colloquia) and activity during classes.

The condition for obtaining class credit is achieving at least 50% of points from each test. The final grade for classes is then determined according to the following scale:

below 50% pts. – fail, [50 – 60%] pts. – satisfactory, [60 – 70%] pts. – satisfactory plus,

[70 – 80%] pts. – good, [80 – 90%] pts. – good plus, [90 – 100%] pts. – very good.

Lecture credit is awarded on the basis of a test during the lecture.

Exam: The condition for taking the examination is obtaining credit for lectures and classes. The condition for passing is achieving at least 50% of the points. The final examination grade is determined according to the following scale:

below 50% pts. – fail, [50 – 60%] pts. – satisfactory, [60 – 70%] pts. – satisfactory plus, [70 – 80%] pts. – good, [80 – 90%] pts. – good plus, [90 – 100%] pts. – very good.

5. Total student workload needed to achieve the intended learning outcomes – number of hours and ECTS credits

Activity	Number of hours
Course hours	60
Other contact hours involving the teacher (consultation hours, examinations)	10
Non-contact hours – student's own work (preparation for classes or examinations, projects, etc.)	80
Total number of hours	150
Total number of ECTS credits	6

* One ECTS point corresponds to 25-30 hours of total student workload

6. Internships related to the course/module

Number of hours	<i>Not applicable</i>
Internship regulations and procedures	<i>Not applicable</i>

7. Instructional materials

<p>Compulsory literature:</p> <ol style="list-style-type: none"> Igor R. Shafarevich, Alexey O. Remizov, Linear Algebra and Geometry, Springer, 2012. P. K. Suetin, Alexandra I. Kostrikin, Yu I Manin, Linear Algebra and Geometry, Taylor & Francis Ltd, 1990. Jim Hefferon, Linear algebra 4th edition, Mathematics and Statistics, Saint Michael's College Colchester, Vermont USA, 2020. https://www.cs.ox.ac.uk/files/12921/book.pdf John M. Erdman, Exercises And Problems In Linear Algebra, World Scientific Publishing Co Pte Ltd, 2020.
<p>Complementary literature:</p> <ol style="list-style-type: none"> Alan Beardon, Algebra and Geometry, Cambridge University, 2005. Nathaniel Johnston, Introduction to Linear and Matrix Algebra, Springer, 2021. Sheldon Axler, Linear Algebra Done Right, fourth edition, Springer, 2024 https://linear.axler.net/LADR4e.pdf Alexandra I. Kostrikin, Exercises in Algebra A Collection of Exercises, in Algebra, Linear Algebra and Geometry, Taylor & Francis Ltd, 1996.

Approved by the Head of the Department or an authorised person