

# **SYLLABUS**

**SUBJECT** Real Analysis (Semester 1)

**TEACHER** Prof. dr hab. Andrzej KAMIŃSKI

## **COURSE DESCRIPTION**

The aim of the course is to provide for the students a short introduction to the theory of measure and integration, important in many areas of mathematics, e.g. in probability theory. The students are expected to understand mathematical notions as well as to use them in practice. An essential extension of the subject of this course are given during the course "Measure Theory" in Semester 3.

The program of the course contains the following: Repetition of elements of the theory of sets and topology of metric spaces. The definition, examples and properties of a  $\sigma$ -algebra of subsets of a given set. Intervals, open sets, closed sets and Borel sets in  $\mathbb{R}$ . The definition, properties and examples of non-negative measures on a  $\sigma$ -algebra of subsets of a given set. The definition, properties and examples of outer measures. The construction of measures via outer measures: the theorem of Caratheodory (without a proof). Complete measures. The definition and properties of the Lebesgue measure in  $\mathbb{R}$ . The definition and properties of measurable functions; Borel functions and Lebesgue measurable functions. The "almost everywhere" identification. Operations on measurable functions. The convergences almost everywhere and in measure. The basic theorems on measurable functions and the convergence of their sequences. The definition and properties of the integral with respect to a nonnegative measure (the Lebesgue integral); characteristic functions, simple functions, integrable functions. Basic convergence theorems: Lebesgue's monotone and dominated convergence theorems. The spaces  $L^p$  and integral inequalities. The product of measures. The Fubini theorem (without a proof). Applications.

**ECTS - 6**

## **LEARNING OUTCOMES**

The examination at the end of the semester will consist of two parts: written and oral exams.

## **GRADING POLICY**

To pass the written exam it is necessary for a student to get more than 50 % of the total possible points. Students who fail the written part still have chance to pass the examination during the oral part. The oral exam is obligatory for all who get not more than 60 % of the total possible points in the written part. Students who get more than 60 % of the total possible points during the written part are released from the oral exam unless they want to improve their grades from the written exam. The grades will be given according to the following rule:

the amount of the received points

in the limits 75.1 % - 100 %	of the total possible points corresponds to the grade	5 (A)
70.1 % - 75.0 %	corresponds to	4.5 (B)
65.1 % - 70.0 %	corresponds to	4 (C)
60.1 % - 65.0 %	corresponds to	3.5 (D)
50.1 % - 60.0 %	corresponds to	3 (E)
0 % - 50.0 %	corresponds to	2 (F)

## **TIMETABLE**

The two-hour lectures will be given on a fixed day every week. The exact time and place will be given later.

**TEXTBOOKS AND REQUIRED MATERIALS**

[1] Avner Friedman,  
*Foundations of Modern Analysis*,  
Dover Publications, New York, the 1970, 1982 or further editions.

[2] Paul R. Halmos,  
*Measure Theory*, Van Nostrand Reinhold, New York, 1950 or further editions.

[3] Stanisław Hartman, Jan Mikusiński,  
*The theory of Lebesgue measure and Integration*, PWN, Warszawa 1957.

[4] Stanisław Łojasiewicz,  
*An Introduction to the Theory of Real Functions*, John Wiley & Sons, Chichester, 1988.

[5] Walter Rudin,  
*Real and Complex Analysis*, McGraw-Hill Book Company, New York, the 1974 or further editions

**PREREQUISITES:**

The knowledge of all courses from the three-year studies of the first degree, in particular of Calculus 1 and 2.