SYLLABUS

SUBJECT Measure Theory (Semester 3)

TEACHER Prof. dr hab. Andrzej KAMIŃSKI

COURSE DESCRIPTION

The aim of the course is to provide for the students an important extension of the theory of measure and integration, shortly presented in the course Real Analysis in Semester 1. The students are expected to understand mathematical notions as well as to use them in practice. The program of the course contains as described below.

Rings, algebras, σ-rings, σ-algebras, monotone families, generated families. Intervals, the generated ring P and algebra Φ in P^d. Open, closed and Borel sets in metric spaces. The definition and properties of non-negative measures on a σ-rings (rings) of subsets of a given set. Finite and σ-finite measures. Generalized distribution functions and a construction of measures on P and Φ. The definition and properties of outer measures. The construction of the outer measure induced by a given measure on a ring. The theorems (with proofs) of Caratheodory, on extending a measure from a ring (algebra) to the generated σ-ring (σ-algebra) and on uniqueness of the extension. Complete measures and a construction of the completion. The definition, properties and characterizations of the Lebesgue measure (measurable sets) in P^d. Nonmeasurable sets. Borel and Lebesgue measurable functions and their properties. Nonmeasurable functions. The convergences almost everywhere and in measure. The theorems of Egoroff, Luzin and Riesz. The definition and properties of the integral with respect to a nonnegative measure (the Lebesgue integral); real- and complex-valued integrable functions; basic convergence theorems (Lebesgue, Fatou). Comparison of the proper and improper Lebesgue and Riemann integrals. The spaces L^p and integral inequalities (Schwarz, Hölder, Minkowski). The definitions and properties of real-valued, complex-valued measures and signed measures, the variation of a measure, the Hahn and Jordan decompositions. Absolute continuity and singularity of measures. The Lebesgue-Radon-Nikodym theorem. The product of measure spaces. The Tonelli and Fubini theorems. Applications. The theory of differentiability, the Lebesgue theorem. Functions of bounded variation. The Lebesgue-Stieltjes integral.

ECTS - 6

LEARNING OUTCOMES

The examination at the end of the semester will consist of two parts: written and oral exams.

GRADING POLICY

To pass the written exam it is necessary for a student to get more than 50 % of the total possible points. Students who fail the written part still have chance to pass the examination during the oral part. The oral exam is obligatory for all who get not more than 60 % of the total possible points in the written part. Students who get more than 60 % of the total possible points during the written part are released from the oral exam unless they want to improve their grades from the written exam. The grades will be given according to the following rule:

the amount of the received points

in the limits 75.1 % - 100 %	of the total possible points	corresponds to the grade	5 (A)
70.1 % - 75.0 %		corresponds to	4.5 (B)
65.1 % - 70.0 %		corresponds to	4 (C)
60.1 % - 65.0 %		corresponds to	3.5 (D)
50.1 % - 60.0 %		corresponds to	3 (E)
0 % - 50.0 %		corresponds to	2 (F)

TIMETABLE

The two-hour lectures will be given on a fixed day every week. The exact time and place will be given later.

TEXTBOOKS AND REQUIRED MATERIALS

Foundations of Modern Analysis, Academic Press, New York, 1960.

[1] Avner Friedman,
Foundations of Modern Analysis,
Dover Publications, New York, the 1970, 1982 or further editions.

[2] Paul R. Halmos,
Measure Theory, D. Van Nostrand, Princeton 1950.

[3] Stanisław Hartman, Jan Mikusiński,
The theory of Lebesgue measure and Integration, PWN, Warszawa 1957.

[4] Stanisław Łojasiewicz,
An Introduction to the Theory of Real Functions, John Wiley & Sons, Chichester, 1988.

[5] Walter Rudin,
Real and Complex Analysis, McGraw-Hill Book Company, New York, the 1974 or further editions

[6] Angus E. Taylor,
General Theory of Functions and Integration, Dover Publications, New York, the 1965, 1985 or further editions.

PREREQUISITES:

The knowledge of the earlier courses from the three-year studies of the first degree and from Semesters 1 and 2 of the two-year studies of the second degree, in particular of the course Real Analysis in Semester 1.