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Concepts and models of nonlinear economic dynamics

INTRODUCTION

Businesses operate in extremely difficult and uncertain economic environments. This is due to numerous factors, especially imperfect market reforms, increasing complexity of the economic structure, a combination of evolution and involution in the process of economic changes. Therefore, there is a need for the usage of nonlinear modelling of economic relationships and processes suitable for the study of possible states and future trends in the economics or specific sectors, markets and so on. Mathematical methods of nonlinear modelling and predictions of economical dynamics should be compact and suitable for the modelling of the current and future state of the studied economical entities.

ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

Investigations into studies on modelling of nonlinear dynamic systems and scientific sources related to the issues of economic dynamics, shows an increasing interest of research into the methods of nonlinear modelling. The first conference of the German Association of Complex Systems and Nonlinear Dynamics (*Deutsche Gesellschaft für Komplexe Systeme und Nichtlineare Dynamik*) took place in Heineberg (Germany) in October 1997. The association was founded a year earlier to promote scientific collaboration between specialists in the areas of natural and fundamental sciences as well as the humanities.

Presently the aforementioned scientific fields of study are based on the theory of nonlinear dynamic systems, especially Synergetics and Chaos Theory. The models and concepts that were developed in synergetics have overlapped the

borders of the pure sciences and have a high heuristic potential¹. The creation of an interdisciplinary structure, according to the vision of its founders, should encourage scientific research of synergistic phenomena in different spheres of life and a description of complex processes of their self-realization from the position of the interdisciplinary approaches. Hermann Haken – a known professor emeritus and chair of theoretical physics at the University of Stuttgart became the honorary chairman of the Association. It was Professor Haken² who in the late 60's introduced the term "synergetics" which later became widespread in the scientific community, particularly in Ukraine. K. Meintser, Head of the Department of Philosophy and Theory of Science at the University of Augsburg, chairs the Association.

The synergistic aspects of the dynamics of natural and social systems are being actively investigated by scientists. Belgian physicist Ilya Prigogine won the Nobel Prize for the development of the theory of irreversible processes and self-organizing systems in nature and society. Murray Gell-Mann, an American Nobel laureate in physics, co-founded and headed the Santa Fe Institute of Complex Adaptive Systems (New Mexico, United States). The Institute carries out research into various issues – from predicting natural disasters to the computer simulation of economic processes.

THE OBJECTIVES OF THIS PAPER

Economic science covers traditional methodology that is related to linear modelling of economic conditions and processes. Linear modelling is an appropriate method for the study of a steady state economy and its individual units in which certain market or national market system tends towards equilibrium. Nevertheless, linear modelling does not provide opportunities for the adequate study of the behavior of an economic system and its issues under dynamic changes, where a steady state and future quantitative and qualitative parameters of economy are uncertain. The research of the non-stationary states that is periodically observed at different stages of the economy, especially during transformations, should be based on non-linear modelling. Nowadays the nonlinear modelling paradigm of economy and theory of its dynamics have begun making inroads into the domestic economic science. The purpose of this article is to show the need and prospects for accelerating this process and to attract the attention of researchers to this issue.

¹ Е.Н. Князева, *Сложные системы и нелинейная динамика в природе и обществе*, http://re-tech.narod.ru/inf/sinergy/ss_nd.htm.

² Г. Хакен, *Синергетика. Иерархии неустойчивостей в самоорганизующихся системах и устройствах*. М.: Мир, 1985. – 419 с.

BRIEF DESCRIPTION OF THE MAIN MATERIAL

The work of the scientists V. Glushkov, V. Leont'yev, Yu. Lysenko, V. Petrenko, L. Serhyeyeva form a methodological basis for economic dynamics modelling. The author believes the time has come for the introduction of a new paradigm, which may become the foundation of modern scientific approach to economic management at all levels. In a more practical sense, there arises an opportunity to explain scientifically the optimal strategy of economic development and forecast possible tendencies and economic disturbances, as well as, show more stable methods for the functioning of an economy and performance of business enterprises.

Until recently, traditional methodology, which is based on mainstream economics, was the dominant approach in economic research. Traditional methodology involves mostly linear modelling of economic relations. It is acceptable for the study of equilibrium (stationary) states of the economy. Linear modelling works properly while considering cases of the achievement of extremum of certain economic parameter – maximum utility function, minimum cost or maximum company's profit. Internal logical completeness and transparency of the linear models add to the economic theories that are based upon them, logical completeness and credibility. In addition, linear tools of economic analysis are extremely easy to use. However, the linear modelling methodology fails to describe adequately the behavior of economic system or its components in unstable conditions.

In terms of synergetics, the static state is undermined, but macroscopic chaoticization (analog to turbulence) is not yet evident. On the basis of the theory of general equilibrium it is impossible to acquire the factors of scientific and technological progress and institutional changes in a country where market transformation is taking place. The approach to the analysis of the economic dynamics, which is based on the theory of self-organization and nonlinear modelling paradigm, creates wider analytical capabilities. It allows one to supplement the traditional theories of economic processes that tend to characterize stationary states (theory of economic statistics) with the theory of economic dynamics (evolutionary economics) related to the behavior of non-stationary open economic systems. In addition, nonlinear modelling methodology is essential for the investigation of evolutionary processes within business enterprises and other economic entities. Under conditions of economic instability and uncertainty which are a feature of economies with ongoing market transformations, e.g. Ukraine, proper analysis of economic processes foresees the combination of both linear and nonlinear modelling.

The integration of nonlinear modelling methodology in economic analysis – initially in natural sciences during the previous decades – preceded the development of the theory of dynamical systems. The theory of dynamical systems was

launched at the beginning of the previous century as the non-linearity of most natural and economic processes became evident. The school of theoretical physics founded by Leonid Mandelstam (1879–1944), focused on the possibility of studying various nonlinear processes. In the late 20's of the XX century Aleksandr Andronov (1901–1952), apprentice of L. Mandelstam, proved that A. Poincaré limit cycles are suitable for the analysis of periodic self-oscillations. Andronov also used the system of the theory of stability that was proposed by A. Lapunov (1857–1918)³ for the analysis of self-oscillations systems. The substantiation of the concept of the dynamic system types by A. Andronov and L. Pontryagin was of great importance for the development of the theory of dynamical systems.

The research program of these prominent scientists was instrumental in distinguishing between “structurally stable” and “structurally unstable” systems. “Structurally unstable system” relate to the subject of bifurcation theory⁴ – a well-developed mathematical science, which is one of the foundations of the theory of nonlinear dynamics. The researchers had at their disposal, the mathematical tools that allowed them to conduct the detailed analysis of dynamical systems with low dimension, thus, they were able to examine issues related to the dynamics of different systems. In the mid-20th century A. Poincaré formed a qualitative mathematical theory was⁵, which allows the analysis of the behavior of dynamic systems by using appropriate equations without seeking their solution. Another important approach in the analysis of dynamic systems is Tikhonov's theorem, which allows one to significantly simplify the system of equations. The application of a set of equations for the specific time intervals is important for the processes of self-organization. This is the subject of synergetics. In some cases, this approach doesn't provide an analytical solution or bifurcations and phase diagrams for the corresponding system of equations. In this case the numerical solution of equations through the use of computers is most suitable.

Nonlinear modelling of economic processes based on the following main postulates⁶ (more information on the following issue is presented in work of W. Zhang⁷):

³ А.М. Ляпунов, *Общая задача об устойчивости движения*. М.: Гостехиздат, 1950.

⁴ А.А. Андронов, Е.А. Леонтович, И.И. Гордон, А.Г. Майер, *Теория бифуркаций динамических систем на плоскости*. М.: Наука, 1967.

⁵ А. Пуанкаре, *О кривых, определяемых дифференциальными уравнениями*. М., Л.: ОГИЗ, 1947.

⁶ Г.Ю. Ризниченко, *Нелинейное естественнонаучное мышление и экологическое сознание // Синергетическая парадигма*. М.: Прогресс-Традиция, 2000, pp. 474–476; Д.А. Козлов, *Методы нелинейной динамики в моделировании макроэкономических процессов*, <http://www.ecfor.ru/pdf.php?id=books/kor001/09>.

⁷ В.Б. Занг, *Синергетическая экономика: Время и переменны в нелинейной экономической теории*. М.: Мир, 1999. – 335 с.

- All processes of natural or social character can be described by nonlinear equations. This is due to the fact that natural and social systems are able to exchange energy and material with the environment and tend to a stable state.
- The nature of the stationary state of the nonlinear system is defined by the type of nonlinearity, system's parameters and its environment, initial conditions of operation.
- System resistance to small deflections is not a common feature because it may occur in a particular phase space (in the chaos theory such spaces are called "strange attractors"), in which the motion of the system becomes unpredictable. In both parametric and phase spaces of the system areas exist where the system is extremely sensitive to even minor fluctuations (perturbations).
- Univocal identification of the parameters of nonlinear systems is mostly impossible. This limits the possibilities of science, the classic task of which is to establish the natural laws of the investigated object by actual data. One can only assume the most important options for development in order to identify a set of probable consequences of certain changes.
- For nonlinear systems the principle of "bottleneck" is not always fair. Therefore, initial nonlinear control base, especially social systems, as opposed to linear, are not yet determined.

General principles of nonlinear modelling behavior of the economic systems and their components are discussed in another work⁸:

- 1) Any economical object is considered to be a point (end of a certain vector), which moves in a multidimensional space by trajectories that can be flat and predictable, or unpredictable and uneven (as happens, for example, during natural disasters);
- 2) The economic system is a multicomponent formation that is capable of self-organization. It is distinguished by the decreasing entropy as seeking to reduce internal chaos by increasing the entropy of its environment, in particular, the consumption of certain resources;
- 3) Functioning of the economic system predicts its reflexive interaction with its environment (similar to the interaction between the financier and the stock exchange);
- 4) The synergistic approach is indispensable to investigate the economic system dynamics on the study. It has to do with learning of the process of self-organizing of dynamic systems existing in nature or society, that's impossible without linear modelling.

The basic mathematical system of nonlinear dynamics is often non-linear differential equations or ordinary derivatives, integral-differential equations,

⁸ Н.В. Катаргин, *Моделирование экономических и социальных систем с использованием нелинейной динамики*, http://www.faito.ru/projects/MatNodEP/Did_Mat/uthpos.pdf.

equations with delay. These equations, unlike ordinary linear differential equations can demonstrate a wide range of possible modes of operation of the dynamic systems. Still some of these equations do not have an analytical solution that is the most valuable in terms of the object of analysis. Therefore, researchers use a number of algorithms for numerical solution of equations and the construction phase and bifurcation diagrams⁹. The appropriateness of the application of such diagrams arises, in particular, in the analysis of economic dynamics in the long run as well as the peculiarities of the behaviour of the economic and other nonlinear systems.

Theorists of economic dynamics have developed many nonlinear models. In developed countries conferences are periodically held and research work on the current subject are published¹⁰.

The first among the elaborated nonlinear models of economic dynamics¹¹ had to do with the possibility of application of the artificial neural networks for technical trading rules (TTR) on market transactions. One model, for example, suitable for selecting TTR is acceptable to predict the nonlinear dynamics of the exchange rate. Although this model cannot be used in relation to long-term time frames, and in some cases also for the final prediction of the price situation on the market.

Another model of economic dynamics¹² serves for identifying regions of stability for the UK economy. The purpose of the developing model is to determine the parameters of the economic system that induce macroeconomic instability. The model is based on a system of differential equations. Analysis of the national economy on the basis of the model contributes to a better understanding of the dynamic properties of the economic system and identify its parameters noted for particularly significant dynamism.

One more model¹³ was proposed by researchers to predict the economic recession and recovery in the different countries. The range of predictable short, medium and long-term economic fluctuations was defined by the construction of the mode. The model allows one to reveal the duration of business trends with

⁹ Ю. Коляда, *Фазові та параметричні портрети ключових математичних моделей нелінійної економ. Динаміки*, http://www.nbu.gov.ua/portal/Soc_Gum/Mise/2010_82/Kolyda.pdf.

¹⁰ "Journal of Economic Theory", 1986. Vol. 40. № 1; "Journal of Economic Theory", 1994. Vol. 63. № 1.

¹¹ P.H. Franses, K. Griensven, *Forecasting Exchange Rates Using Neural Networks for Technical Trading Rules* // "Studies in Nonlinear Dynamics and Econometrics Quarterly Journal, January", 1998. Vol. 2, № 4.

¹² W. Barnett, He. Yijun, *Stability Analysis of Continuous-Time Macroeconometric Systems* // "Studies in Nonlinear Dynamics and Econometrics Quarterly Journal", 1999. Vol. 3. №4.

¹³ A. Garcia-Ferrer, A. Ricardo, *Queral Using Long-, Medium-, and Short-Term Trends to Forecast Turning Points in the Business Cycle: Some International Evidence* // "Studies in Nonlinear Dynamics and Econometrics Quarterly Journal", July. 1998. Vol. 3. № 2.

specific properties. The practical value of the model lies in the fact that by certain adjustment it is acceptable to predict the critical point of business cycle in the past period of time as well as in the future. The model was applied conformably to quarterly variables for the United States in the post-war period, as well as a number of variables for some Western European countries.

In recent years several nonlinear models of macroeconomic dynamics has been proposed by Russian researchers A. Smirnov and D. Chernavsky. Model A. Smirnova¹⁴ is distinguished by the use of economic dynamics to analyse the elements of the theory of catastrophes. Scientists also proposed constructing a nonlinear model algorithm for the market transformation of the economic system. The scientific advantage of the approach is the author's classification of economic systems as a kind of command and market economy and to demonstrate the process of building a market economy, such as "an assembling" of its components together. However, analysis of the transformational changes concerns the retrospective approach. The model is not suitable for predicting the process of the market transformation of the economy and possible trajectories of dynamics in the future. In another researcher's work¹⁵ attempts were made to overcome these weaknesses of the model through predictive modelling of the future dynamics of the Russian economy. Based on the methods of nonlinear dynamics, the scientist explained different trajectories of the future development. A. Smirnov gave arguments for such conclusion because probable forecasts of economic dynamics, even if they are supported by substantial empirical evidence, may mark a high level of uncertainty due to inaccuracies in the formulation of scientific hypotheses or contain logical contradictions. The scientist also emphasized the need to identify the key parameters in the development of the economic system and to take into account not only economic but also political processes.

A model of economic dynamics was proposed by D. Chernavsky¹⁶, based on the principle of the needs of the national economy emphasizes power and parametric methods of "switching" of economic development, designed to guide the economic dynamics in the direction, which researchers characterize as a "highly productive state" (Figure 1). The methodological value of the model lies in taking into account features of a transition economy that are characterized by a high level of instability and uncertainty. In addition, the model is one of the possible approaches for determination of the most suitable option among various alternatives of macroeconomic policy.

¹⁴ А.Д. Смирнов, *Модель социально-экономической перестройки* // Эконом. и мат. методы. 1992. Т. 28. Вып. 2.

¹⁵ А.Д. Смирнов, *Переходная экономика: модель прогноза* // Проблемы прогнозирования, 1993. №2.

¹⁶ Д.С. Чернавский, Н.И. Старков, А.В. Щербаков, *Динамическая модель закрытого общества (институциональные ловушки и кризисы)* // Математическое моделирование. 2001. Т. 13. №11.

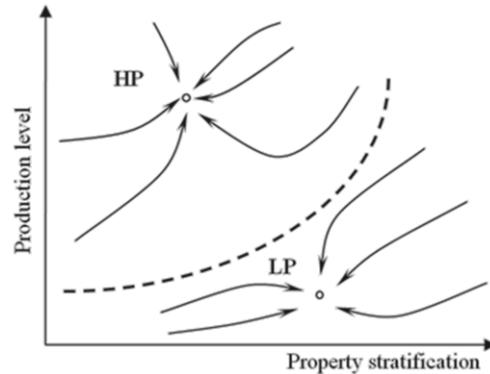


Figure 1. Possible movement pattern of transitive economy

HP – highly-productive condition; LP – low productive condition

Source: С.Ю. Малков, *Нелинейная динамика нелинейного мира*, <http://nonlin.ru/node/902>.

All of these approaches are related to the nonlinear modelling of economic dynamics and were proposed by researchers to explain or predict certain micro – and macro-economic tendencies, and take actions concerting economic policy, which are aimed to limit impact on economic development of the unfavourable disturbance as much as possible.

In addition to these models, we provide examples of some simple models in mathematical formulation. It appears that co-evolutionary processes in the world economy can be described with the help of the simple non-linear models¹⁷:

$$\begin{aligned}\frac{dx}{dt} &= ax^2 - c(x - y), \\ \frac{dy}{dt} &= by^2 + c(x - y), \\ x(0) &= x_0, y(0) = y_0,\end{aligned}$$

where:

x, y – Gross Domestic Product per capita of two interacting countries,

c – coefficient characterizing the intensity of the interaction of national economies,

x_0, y_0 – the initial conditions ($x_0 > y_0$), x relates to the leader country,

y – the country that overtakes it.

Robert Solow's model is a single-section model of a closed economy. In this kind of model, the economic system is considered as a whole economic entity that produces only one product that can be consumed or invested. The model adequately enough describes the major macroeconomic aspects of reconstruc-

¹⁷ С.Ю. Малков, *Нелинейная динамика...*

tion. In this model export and import of goods are not taken into account¹⁸. Using relative ratio of national production the following model can be displayed with the help of such equations set:

$$\frac{dk}{dt} = -\lambda k + \rho(1-a)f(k), \quad \lambda = \mu + \nu, \quad k(0) = k_0 = \frac{K_0}{L_0};$$

$$x = f(k);$$

$$i = \rho(1-a)f(k);$$

$$c = (1-\rho)(1-a)f(k).$$

After receiving the model of economic process, the problem of the identification of parameters arises. Mostly parameters are determined by using statistical methods. In the case of nondimensionalization, the analysed models often contain small parameters in the right side of equations (regular perturbation), or by derivatives (singular perturbation). In the mathematical science many asymptotic methods¹⁹ are also developed, which can be successfully applied in the study of nonlinear models of economic dynamics.

CONCLUSIONS

Economic development occurs under conditions of instability and uncertainty which offers possibilities for diverse interpretations. Therefore, the need for the determination of the effective alternative of economic development and its business units in the current environment and in the future is updated on the basis of the concept of evolutionary economics and nonlinear modelling of economic dynamics methodology.

The abovementioned approaches of economic dynamics modelling are related to co-evolutionary processes, which are associated with economic integration and possible forecasts as to quantitative parameters of a closed economy. Further development of the theory of economic dynamics is possible by means of nonlinear modelling of impact on production and general economic dynamics of the human development factors and stable economic growth²⁰. The modelling of economic dynamics with changes in the institutional structure of the economy is also of special importance.

¹⁸ В.В. Вітлінський, *Моделювання економіки*. К.: КНЕУ, 2003.– 408 с; В.А. Колемаев, *Математическая экономика*. М.: ЮНИТИ, 1998. – 240 с.

¹⁹ А.Б. Васильева, В.Ф. Бутузов, *Асимптотические разложения решений сингулярно возмущенных уравнений*. М.: Наука, 1973. – 272 с; Н.Н. Моисеев, *Асимптотические методы нелинейной механики*. М.: Наука, 1969 – 379 с; А.Х. Найфе, *Методы возмущений*. М.: Мир, 1976 – 456 с; I.L. Lions, *Perturbations singulières dans les problèmes aux limites et en contrôle optimal*. //Lect. Notes Math., 323, 12, 1973. – 540, p. 25.

²⁰ Т. Вінод., *Якість економічного зростання*. К.: Основи, 2002, – 350 с. – с. 23, С. 33–49.

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Summary

Economic development which occurs under conditions of instability and uncertainty give rise to the possibility of diverse ways of its further elaboration. Therefore, the need for determination of the effective alternative of economic development and its business units in the current environment and in the future is updated on the basis of the concept of evolutionary economics and nonlinear modelling of economic dynamics methodology. The proposed approaches to the modelling of economic dynamics are interrelated with co-evolutionary processes, which are associated with economic integration, and possible forecasts as to quantitative parameters of a closed economy. The article is devoted to the development of the theory of economic dynamics by means of nonlinear modelling impact on production and general economic dynamics of the human development factors and stable economic growth.

Koncepcje i modele dynamiki gospodarczej

Streszczenie

Rozwój gospodarczy odbywa się w warunkach niestabilności i niepewności, co określa możliwość różnych sposobów jego dalszego rozwoju. Dlatego aktualna jest potrzeba w zdefiniowaniu racjonalnego wariantu rozwoju gospodarczego jej, jednostek biznesowych tak w obecnej sytuacji, jak i w przyszłości – na podstawie koncepcji ekonomii ewolucyjnej i metodologii modelowania nieliniowego dynamiki gospodarczej. Te podejścia do modelowania dynamiki gospodarczej dotyczą ewolucyjnych procesów związanych z integracją gospodarczą i możliwych prognoz dotyczących ilościowych parametrów rozwoju gospodarki zamkniętej. Artykuł jest poświęcony teorii dynamiki gospodarczej za pomocą modelowania nieliniowego wpływu na produkcję i teorii ogólnej dynamiki czynników rozwoju społecznego oraz trwałego wzrostu gospodarczego.